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Strongly Interacting Longitudinal Gauge Bosons In Supersymmetric Models

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Abstract

A non-linear sigma model effective lagrangian is analyzed for theories in which supersymmetry is softly broken at scales below the electroweak symmetry breaking scale. Besides the gauge and matter supermultiplets, the low energy theory contains only three Goldstone chiral multiplets. The higgsino, gaugino as well as the charged and neutral Higgs bosons have (light) phenomenologically acceptable masses, the values of which depend on the explicit soft supersymmetry breaking parameters. In addition, the longitudinal vector bosons become strongly interacting at high energies ($M_Z \ll E \ll 4\pi v$). The equivalence theorem is exploited in order to obtain their scattering amplitudes. Furthermore, supersymmetry results in enhanced longitudinal vector boson production of Higgs bosons.

1 Introduction

The minimal supersymmetric standard model (MSSM) [1][2][3] compactly solves the naturalness as well as the technical fine-tuning problems. The electroweak symmetry breaking is catalyzed by the soft (no quadratic divergence) supersymmetry (SUSY) breaking terms arising from the hidden supergravity sector. As a consequence, the Higgs sector self-couplings are given by the electroweak gauge coupling constants. Hence, the model remains perturbative up to the Planck scale. Generalizations of the MSSM, whether motivated from string inspired grand unified theories or from attempts to solve the μ -problem, involve additional electroweak matter multiplets. In particular, the new multiplets mix with the two Higgs superfields of the MSSM so as to alter the mass spectrum. The paradigm for such a process is the minimal plus singlet supersymmetric standard model, (M+1)SSM [4][5]. The tree level upper bound of the lightest neutral scalar is raised from M_Z to a bound that depends on the singlet-doublet Higgs interaction strength, g , and the ratio of Higgs doublet vacuum expectation values, $\tan \beta \equiv \frac{v_B}{v_T}$,

$$M_h^2 = M_Z^2 \left(\cos^2 2\beta + \frac{2g^2}{g_1^2 + g_2^2} \sin^2 2\beta \right).$$

In this case, as in the MSSM, the internal symmetries are to remain perturbatively natural up to the GUT scale. The coupling constant g has an infrared quasi-fixed point value of approximately 0.87 [6], which leads to a lightest Higgs mass upper bound of around 120 GeV [7][8][9][10][11][12].

More complex extensions of the MSSM follow patterns similar to that of the (M+1)SSM [13][14]. Detailed parameterizations of the mass spectrum and the soft SUSY breaking parameter values have been addressed in numerous studies. In all cases the parameter space is investigated that provides a

SUSY breaking scale above that of the electroweak scale and, more specifically, a perturbative mechanism for electroweak symmetry breaking that remains such up to GUT scales. In the MSSM there is no alternative, it is a prescribed part of the model. In the (M+1)SSM, it is the *raison d'être* for the SUSY model, but as such, a matter of choice. In this paper we desire to study the situation in which SUSY remains unbroken at energies below the electroweak symmetry breaking scale, $\Lambda = 4\pi v$, with $v \approx 250$ GeV. Since, in the broken symmetry phase, only the partners of the electroweak Goldstone bosons need be light, we will be considering the heavy mass (triviality) limit for the remaining particles. The low energy effective theory consequently contains fewer SUSY multiplets of particles than the perturbative MSSM. In the (M+1)SSM, this means that the entire singlet as well as the heavy neutral Higgs, the pseudoscalar and their fermion partner become more massive than Λ . Without soft SUSY breaking the partners to the Goldstone bosons, that is the remaining light neutral Higgs and the two charged Higgs particles, are degenerate in mass with the Z^0 and the W^\pm , respectively, as are their fermionic partners according to the SUSY Higgs mechanism. With soft SUSY breaking, the light neutral Higgs field can acquire a mass higher than M_Z and the charged Higgs fields a mass higher than M_W . The fermion partners along with the corresponding gauginos, however, will acquire masses half of which are lower and half of which are higher than that of the gauge bosons.

Of course, in lowest order, the low energy dynamics described above is independent of the particular short distance physics that gives rise to it, be it a strongly interacting (M+1)SSM or some complicated dynamical symmetry breaking scheme, as long as SUSY is softly broken at scales $M_{SUSY} \leq \Lambda$.

Hence, the corresponding effective (softly broken) supersymmetric action is the same, regardless of the mechanism for electroweak symmetry breaking. Thus the characteristic mass spectrum for every such SUSY model will have, besides massless Goldstone bosons, neutral and charged Higgs particles with masses below $\Lambda = 4\pi v$. In addition there will be neutral and charged fermion partners with some masses in the M_Z or M_W to Λ range and an equal number with masses in the 0 to M_Z or M_W range, respectively. The exact values, as will be seen, depend on the explicit values of the SUSY breaking parameters.

In the non-supersymmetric case, recall that the heavy Higgs limit of the standard model can be described by the strong Higgs self-coupling limit of the scalar fields. Indeed, the scalar sector of the standard model can be parameterized by the unitary 2×2 matrix

$$\begin{aligned} U &\equiv \begin{pmatrix} (\tilde{\phi}) & (\phi) \end{pmatrix} \\ &= \begin{pmatrix} h^{0\dagger} & h^+ \\ -h^- & h^0 \end{pmatrix} \end{aligned} \tag{1.1}$$

with

$$\phi = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \tag{1.2}$$

the usual Higgs doublet field and

$$\tilde{\phi} = \begin{pmatrix} h^{0\dagger} \\ -h^- \end{pmatrix} \tag{1.3}$$

its hypercharge conjugate doublet. The scalar sector of the standard model has the gauge invariant Lagrangian

$$\mathcal{L} = \frac{1}{4} \text{Tr} [D_\mu U^\dagger D^\mu U] - \frac{\lambda}{4} \left[\frac{1}{2} \text{Tr} [U^\dagger U] - v^2 \right]^2, \tag{1.4}$$

with the gauge covariant derivative

$$D_\mu U = \partial_\mu U + ig_2 \vec{W}_\mu \cdot \vec{T} U - ig_1 B_\mu U T^3, \quad (1.5)$$

where $\vec{T} = \frac{1}{2}\vec{\sigma}$ and $\vec{\sigma}$ are the Pauli matrices. The strong coupling limit, $\lambda = \frac{1}{2}(\Lambda/v)^2 \gg 1$, leads to the constraint on the scalar fields $\frac{1}{2}\text{Tr}[U^\dagger U] = v^2$ with the interpretation that only the Goldstone bosons remain in the theory at energies below the electroweak symmetry breaking scale $\Lambda = 4\pi v$ and the gauge symmetry transformations are realized non-linearly. Radiative corrections to this tree effective lagrangian can be included by considering $\text{Tr}[D_\mu U^\dagger D^\mu U]$ to be the lowest order term in a derivative expansion of the effective lagrangian for momenta below Λ . In references [15]-[18] detailed lists of all dimension four through six operators are given, including the lowest order custodial $SU(2)$ symmetry violating term $\text{Tr}[T^3 D_\mu U^\dagger D^\mu U]$ whose coefficient describes the contributions of the gauge radiative corrections to $\Delta\rho$, as well as the new physics above Λ .

The resulting derivatively coupled non-linear σ -model effective lagrangian describes strongly self-interacting Goldstone bosons at high energy. According to the equivalence theorem [19][20][21][22], the scattering amplitudes involving longitudinally polarized gauge bosons are equivalent, up to corrections of $O(\frac{M_W}{E})$, to the amplitudes with the longitudinal gauge bosons, W_L^\pm , Z_L replaced by the corresponding Goldstone bosons, w^\pm , z , at high energies

$$T(W_L, \dots, Z_L, \dots) = T(w, \dots, z, \dots) + O(\frac{M_W}{E}). \quad (1.6)$$

Hence, these tree longitudinal gauge boson scattering amplitudes at high energy can be read off directly from the non-linear Goldstone boson lagrangian. In the notation of an invariant length interval in the symmetric

space $SU(2) \times SU(2)/SU(2)$ [23][24], the lagrangian is

$$\mathcal{L} = \frac{1}{2} \partial_\mu \pi^i g_{ij}(\pi) \partial^\mu \pi^j, \quad (1.7)$$

with the choice of coordinates $U = \sigma \mathbf{1} + 2i\vec{T} \cdot \vec{\pi}$ and with the constraint $\frac{1}{2}\text{Tr}[U^\dagger U] = \det U = v^2$ yielding $\sigma = \sqrt{v^2 - \vec{\pi}^2}$ (i.e. $h^0 = \sigma - i\pi^3$ and $h^+ = i(\pi^1 - i\pi^2)$). This yields the metric for this parametrization of U : $g_{ij} = \delta_{ij} + \pi^i \pi^j / (v^2 - \vec{\pi}^2)$. When expanded in a power series in π^i/v , the enhanced longitudinal gauge boson scattering amplitudes at high energy, $E \gg M_W$, are simply obtained [22][25]

$$\begin{aligned} T(W_L^+ W_L^- \longrightarrow Z_L Z_L) &\simeq i \frac{s}{v^2} \\ T(W_L^+ W_L^- \longrightarrow W_L^+ W_L^-) &\simeq -i \frac{u}{v^2}, \end{aligned} \quad (1.8)$$

while

$$T(Z_L Z_L \longrightarrow Z_L Z_L) \simeq 0. \quad (1.9)$$

The remaining longitudinal gauge boson amplitudes can be obtained by crossing symmetry, for example

$$T(W_L^\pm Z_L \longrightarrow W_L^\pm Z_L) \simeq i \frac{t}{v^2}. \quad (1.10)$$

Hence, we are led to the optimistic alternatives of a light (perturbative) Higgs boson ($M_h < \Lambda$) being observed directly, or, if heavy ($M_h > \Lambda$), the enhanced scattering of longitudinal gauge bosons.

Analogously, the Kähler potential describing the non-linear realization of the supersymmetric $SU(2) \times U(1)$ electroweak gauge symmetry can be written in terms of the two Higgs doublet chiral superfields (recall that σ and $\vec{\pi}$ are chiral superfields here)

$$H_B = \begin{pmatrix} H^+ \\ H_B^0 \end{pmatrix}$$

$$= \begin{pmatrix} i\pi^+ \\ \sigma - i\pi^3 \end{pmatrix} \quad (1.11)$$

and

$$\begin{aligned} H_T &= \begin{pmatrix} H_T^0 \\ H^- \end{pmatrix} \\ &= \begin{pmatrix} \sigma + i\pi^3 \\ i\pi^- \end{pmatrix}, \end{aligned} \quad (1.12)$$

grouped to form the matrix chiral superfield U

$$\begin{aligned} U &= \sigma \mathbf{1} + 2i\vec{T} \cdot \vec{\pi} \\ &= \begin{pmatrix} (H_T) & (H_B) \end{pmatrix} \\ &= \begin{pmatrix} H_T^0 & H^+ \\ H^- & H_B^0 \end{pmatrix} = \begin{pmatrix} \sigma + i\pi^3 & i(\pi^1 - i\pi^2) \\ i(\pi^1 + i\pi^2) & \sigma - i\pi^3 \end{pmatrix}. \end{aligned} \quad (1.13)$$

The matrix is constrained,

$$\det U = H_T \epsilon H_B = \sigma^2 + \vec{\pi}^2 = \frac{1}{2} v_T v_B = \frac{1}{4} v^2 \sin 2\beta, \quad (1.14)$$

so that a non-linear realization of $SU(2) \times U(1)$ is induced on $\vec{\pi}$

$$U' = LUR^{-1}, \quad (1.15)$$

with $L = e^{i\vec{T} \cdot \vec{\Lambda}}$ and $R = e^{iT^3 \Lambda_Y}$, where the chiral superfields $\vec{\Lambda} (\Lambda_Y)$ parameterize the $SU(2) (U(1))$ gauge transformations. The gauge invariant Kähler potential is made from powers of the two independent $SU(2) \times U(1)$ gauge invariant superfields [26]

$$\begin{aligned} X &= \text{Tr} \left[\bar{U} e^{-2g_2 \vec{T} \cdot \vec{W}} U e^{2g_1 T^3 Y} \right] \\ Y &= \text{Tr} \left[T^3 \bar{U} e^{-2g_2 \vec{T} \cdot \vec{W}} U e^{2g_1 T^3 Y} \right], \end{aligned} \quad (1.16)$$

where \vec{W} and Y are the $SU(2)$ and $U(1)$ gauge fields, respectively. Hence the most general Kähler potential is given by

$$K = \sum_{m,n=0}^{\infty} K_{mn} X^m Y^n, \quad (1.17)$$

which yields the lowest order terms in a derivative expansion of the action, $\Gamma = \int dV K$. Note that $\Gamma = \int dV X$ is the simplest such action. As well it has the form of the MSSM Higgs fields' kinetic energy and so preserves $\rho = 1$ at the tree level. Indeed, Y will involve violations of $\rho = 1$, as can be seen most easily in the unitary gauge (here $v_T = v_B$ and SUSY is unbroken in order of simplify, inessentially, the algebra)

$$\begin{aligned} X|_{Unitary} &= v^2 \left[1 + g_2^2 W^+ W^- + \frac{1}{2}(g_2^2 + g_1^2) Z^2 \right] \\ Y|_{Unitary} &= -v^2 \sqrt{g_2^2 + g_1^2} Z. \end{aligned} \quad (1.18)$$

So, for instance, the Y^2 term in K yields a non-trivial $\Delta\rho$: $\Delta\rho \sim K_{02}$.

In section 2 we will describe the non-linear sigma model effective action for supersymmetric electroweak symmetry breaking. The mass spectrum in the SUSY Higgs and gaugino sector is fully determined in terms of the soft SUSY breaking parameters and M_Z and M_W . At the tree level the gauge field and (s)matter sector masses are as in the MSSM. In section 3 the equivalence theorem relating the scattering amplitudes for longitudinal vector bosons to the corresponding Goldstone boson amplitudes is exploited. Although the presence of light Higgs fields is possible, the longitudinal Goldstone bosons are still strongly interacting [27]. The tree amplitudes for longitudinal W^\pm and Z^0 scattering are shown to grow with energy:

$$\begin{aligned} T(W_L^+ W_L^- \longrightarrow Z_L Z_L) &\simeq i \sin^2 2\beta \frac{s}{v^2} \\ T(W_L^+ W_L^- \longrightarrow W_L^+ W_L^-) &\simeq -i \sin^2 2\beta \frac{u}{v^2}, \end{aligned} \quad (1.19)$$

while

$$T(Z_L Z_L \longrightarrow Z_L Z_L) \simeq 0. \quad (1.20)$$

Since SUSY is broken at comparable scales, it is found that the scattering of longitudinal gauge bosons into neutral and charged Higgs bosons is similarly enhanced. In fact, the scattering amplitude for longitudinal Z^0 bosons to produce light Higgs bosons, $Z_L Z_L \longrightarrow h h$, is the dominant mode to neutrals since, for $s > M_h^2$, the amplitude for $Z_L Z_L \longrightarrow Z_L Z_L$ vanishes, as noted, while the Higgs production amplitudes grow with s

$$\begin{aligned} T(Z_L Z_L \longrightarrow h h) &\simeq -i \sin^2 2\beta \frac{2s}{v^2} \\ T(W_L^+ W_L^- \longrightarrow h h) &\simeq -i \sin^2 2\beta \frac{s}{v^2}. \end{aligned} \quad (1.21)$$

The appendices recall some facts about non-linear realizations of gauge symmetries in supersymmetric theories. In appendix A the heavy Higgs limit of the (M+1)SSM is shown to lead to the supersymmetric non-linear sigma model with a chiral Goldstone superfield for each broken generator. The explicit form of the component field non-linear sigma model lagrangian is derived in appendix B. The Killing vectors in different coordinate systems for the $SU(2) \times U(1)/U(1)$ Kähler manifold are discussed in appendix C.

2 The Effective Action

The Kähler manifold describing the non-linear realization of spontaneously broken electroweak symmetry breaking in supersymmetric theories has the chiral superfield coordinates π^i , $i = 1, 2, 3$. (We will deal with the completely doubled realization [28] of the electroweak gauge symmetry; see reference [29] for a discussion of the possibility of a minimal realization with non-trivial fixed points.) As suggested in the introduction, a specific coordinate system can be chosen by taking the heavy Higgs limit of the (M+1)SSM (see appendix A), although all the low energy physics is re-parameterization invariant and so it does not depend on this specific choice. Hence, we introduce the Goldstone chiral supermultiplets

$$\begin{aligned} H_B &\equiv \begin{pmatrix} H^+ \\ H_B^0 \end{pmatrix} = \begin{pmatrix} i\pi^+ \\ \sigma - i\pi^0 \end{pmatrix} \\ H_T &\equiv \begin{pmatrix} H_T^0 \\ H^- \end{pmatrix} = \begin{pmatrix} \sigma + i\pi^0 \\ i\pi^- \end{pmatrix}, \end{aligned} \quad (2.1)$$

with

$$\begin{aligned} \pi^\pm &= \pi^1 \mp i\pi^2 \\ \sigma &= \sqrt{\frac{1}{2}v_T v_B - \vec{\pi}^2} = v\sqrt{\frac{1}{4}\sin 2\beta - \vec{\pi}^2/v^2}, \end{aligned} \quad (2.2)$$

where the vacuum values of the supermultiplets are given by

$$\begin{aligned} \langle H_B \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_B \end{pmatrix} \\ \langle H_T \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_T \\ 0 \end{pmatrix} \end{aligned} \quad (2.3)$$

and the ratio of vacuum values defines the angle β through $\tan \beta = v_B/v_T$ and the electroweak vacuum value $v^2 = v_T^2 + v_B^2$. The electroweak gauge transformations have the usual form for the chiral superfields

$$\begin{aligned} H'_B &= e^{\frac{i}{2}\Lambda_Y} e^{i\vec{\Lambda}\cdot\vec{T}} H_B \\ H'_T &= e^{-\frac{i}{2}\Lambda_Y} e^{i\vec{\Lambda}\cdot\vec{T}} H_T. \end{aligned} \quad (2.4)$$

Due to the above constraint, $H_T \epsilon H_B = \sigma^2 + \vec{\pi}^2 = \frac{1}{2}v_T v_B$, this is actually a non-linear realization on the Goldstone superfields π^i . The $SU(2)$ gauge fields, $W^i, i = 1, 2, 3$, and the $U(1)$ gauge field Y , transform as

$$\begin{aligned} e^{-2g_2\vec{W}'\cdot\vec{T}} &= e^{i\vec{\Lambda}\cdot\vec{T}} e^{-2g_2\vec{W}\cdot\vec{T}} e^{-i\vec{\Lambda}\cdot\vec{T}} \\ e^{-g_1 Y'} &= e^{\frac{i}{2}\vec{\Lambda}_Y} e^{-g_1 Y} e^{-\frac{i}{2}\Lambda_Y}. \end{aligned} \quad (2.5)$$

The remaining quark and lepton matter superfields transform as in the MSSM, see appendix B. Defining the combination of electroweak gauge fields

$$\begin{aligned} V_B &= \frac{1}{2}g_1 Y + g_2 \vec{T} \cdot \vec{W} \\ V_T &= -\frac{1}{2}g_1 Y + g_2 \vec{T} \cdot \vec{W}, \end{aligned} \quad (2.6)$$

the fundamental gauge invariant Goldstone superfield terms are given by $(\bar{H}_B e^{-2V_B} H_B)$ and $(\bar{H}_T e^{-2V_T} H_T)$. The remaining matter field kinetic energy and Yukawa terms are as in the MSSM and so are relegated to Appendix B.

The simplest lowest order Goldstone superfield effective action, $\Gamma = \int dV K$, is given by the Kähler potential

$$K = \bar{H}_T e^{-2V_T} H_T + \bar{H}_B e^{-2V_B} H_B. \quad (2.7)$$

The most general soft SUSY breaking terms [30] (soft in that only logarithmic corrections in the momentum Λ occur) are the $\rho = 1$ preserving, $\theta \bar{\theta}$

independent terms from K and the $\theta\bar{\theta}$ independent, $\Delta\rho$ producing terms from $(\bar{H}_T e^{-2V_T} H_T - \bar{H}_B e^{-2V_B} H_B)$. Hence the gauge invariant Goldstone superfield action can be written as

$$\begin{aligned}\Gamma_G &= \int dV [1 + a\theta^2\bar{\theta}^2]K \\ &\quad + \int dV b\theta^2\bar{\theta}^2 [\bar{H}_T e^{-2V_T} H_T - \bar{H}_B e^{-2V_B} H_B],\end{aligned}\quad (2.8)$$

with a and b the only SUSY breaking parameters in the pure Goldstone sector. When the gaugino soft SUSY breaking mass terms and the Yang-Mills auxiliary field terms are included, the vacuum and the Goldstone bosons', electroweak gauge bosons' and their superpartners' mass spectra can be determined. In components the superfield Kähler potential action, including the gaugino mass lagrangian, $\mathcal{L}_{YM\hat{\phi}}$, and Yang-Mills auxiliary D_T field and D_B field lagrangian terms, \mathcal{L}_{YMD} , reduces to the usual Kähler form of the Higgs SUSY Lagrangian (see Appendix B):

$$\Gamma_K = \int d^4x \mathcal{L}_K \quad (2.9)$$

where

$$\mathcal{L}_K = \mathcal{L}_{GS} + \mathcal{L}_{G\hat{\phi}} + \mathcal{L}_{YMD} + \mathcal{L}_{YM\hat{\phi}} \quad (2.10)$$

with

$$\Gamma_G = \int dx (\mathcal{L}_{GS} + \mathcal{L}_{G\hat{\phi}}) \quad (2.11)$$

and

$$\begin{aligned}\mathcal{L}_{GS} &= (D_\lambda A_T)^\dagger (D^\lambda A_T) + i\bar{\psi}_T \bar{D} \psi_T + F_T^\dagger F_T \\ &\quad - A_T^\dagger D_T A_T + \sqrt{2} [\bar{\psi}_T \bar{\lambda}_T A_T + A_T^\dagger \lambda_T \psi_T] + (T \longrightarrow B).\end{aligned}\quad (2.12)$$

The generic chiral superfield in components is given by

$$H = e^{-i\theta\phi\bar{\theta}} [A - i\sqrt{2}\theta^\alpha\psi_\alpha + \theta^2 F], \quad (2.13)$$

where the complex first component has the general real pseudoscalar field, P , and real scalar field, S , structure $A = P - iS$. The soft SUSY breaking terms for the Goldstone multiplets from equation (2.8) define \mathcal{L}_{G^\sharp} , and are simply given by the component field lagrangian

$$\mathcal{L}_{G^\sharp} = (a + b)A_T^\dagger A_T + (a - b)A_B^\dagger A_B. \quad (2.14)$$

The auxiliary gauge field and the gaugino soft SUSY breaking terms from the Yang-Mills sector are also considered in the calculation of the vacuum state and Goldstone and gauge supermultiplet masses. The corresponding Lagrangian for the auxiliary fields is

$$\mathcal{L}_{YMD} = -\frac{1}{2}\vec{D}_W \cdot \vec{D}_W - \frac{1}{2}D_Y^2, \quad (2.15)$$

while the gaugino mass SUSY breaking terms are

$$\begin{aligned} \mathcal{L}_{YM^\sharp} &= \frac{1}{2}\tilde{m}_W \vec{\lambda}_W \cdot \vec{\lambda}_W + \frac{1}{2}\tilde{m}_Y \lambda_Y^2 + h.c. \\ &= \frac{1}{2} \begin{pmatrix} \lambda_\gamma & \lambda_Z \end{pmatrix} \begin{pmatrix} \tilde{m}_\gamma & \tilde{m}_{\gamma Z} \\ \tilde{m}_{\gamma Z} & \tilde{m}_Z \end{pmatrix} \begin{pmatrix} \lambda_\gamma \\ \lambda_Z \end{pmatrix} + \tilde{m}_W \lambda_+ \lambda_- + h.c. \end{aligned} \quad (2.16)$$

The mass and electroweak gaugino eigenfields are related, as usual, via the weak mixing angle θ_W with $\tan \theta_W = g_1/g_2$ and charged fields $\lambda_\pm = \frac{1}{\sqrt{2}}(\lambda_1 \mp i\lambda_2)$

$$\begin{aligned} \lambda_\gamma &= \lambda_3 \sin \theta_W + \lambda_Y \cos \theta_W \\ \lambda_Z &= \lambda_3 \cos \theta_W - \lambda_Y \sin \theta_W, \end{aligned} \quad (2.17)$$

so that the soft SUSY breaking gaugino masses are related by

$$\begin{aligned} \tilde{m}_\gamma &= (\tilde{m}_W \sin^2 \theta_W + \tilde{m}_Y \cos^2 \theta_W) \\ \tilde{m}_Z &= (\tilde{m}_W \cos^2 \theta_W + \tilde{m}_Y \sin^2 \theta_W) \\ \tilde{m}_{\gamma Z} &= \frac{1}{2}(\tilde{m}_W - \tilde{m}_Y) \sin 2\theta_W. \end{aligned} \quad (2.18)$$

The masses of the Goldstone-Higgs boson and gaugino-higgsino sectors can be determined from above. Eliminating the constrained fields and shifting the Higgs fields by their vacuum expectation values, the kinetic energy terms for the neutral Higgs and higgsino fields acquire a finite wavefunction renormalization factor (see Appendix B). Rescaling the ψ_3 field by this factor,

$$\psi_3 \longrightarrow \frac{1}{2}\sqrt{1 + \sin 2\beta} \psi_3, \quad (2.19)$$

the neutral fermion mass matrix in the $(\lambda_Z, \psi_3, \lambda_\gamma)$ basis becomes

$$\tilde{M} = \begin{pmatrix} \tilde{m}_Z & M_Z & \tilde{m}_{\gamma Z} \\ M_Z & 0 & 0 \\ \tilde{m}_{\gamma Z} & 0 & \tilde{m}_\gamma \end{pmatrix}, \quad (2.20)$$

while the charged fermion mass matrix in the complex basis (λ_+, ψ_+) is

$$\tilde{M}_{ch} = \begin{pmatrix} \tilde{m}_W & M_W \sqrt{2} \cos \beta \\ M_W \sqrt{2} \sin \beta & 0 \end{pmatrix}. \quad (2.21)$$

The gauge boson masses are found to be given by their usual form

$$\begin{aligned} M_W &= \frac{1}{2} g_2 v \\ M_Z &= \frac{1}{2} \sqrt{g_1^2 + g_2^2} v. \end{aligned} \quad (2.22)$$

The neutralino matrix is Hermitian and can be diagonalized directly. For simplicity we choose $\tilde{m}_{\gamma Z} = 0$, implying that $\tilde{m}_W = \tilde{m}_Y$ and hence $\tilde{m}_Z = \tilde{m}_W = \tilde{m}_\gamma$, the photino mass, \tilde{M}_γ , is simply given by the breaking term $\tilde{M}_\gamma = \tilde{m}_\gamma$. The squares of the zino mass, \tilde{M}_Z , and the higgsino mass, \tilde{M}_h are

$$\begin{aligned} \tilde{M}_Z^2 &= M_Z^2 \left[1 + \frac{1}{2} \frac{\tilde{m}_Z^2}{M_Z^2} \left(\sqrt{1 + 4 \frac{M_Z^2}{\tilde{m}_Z^2}} + 1 \right) \right] \\ \tilde{M}_h^2 &= M_Z^2 \left[1 - \frac{1}{2} \frac{\tilde{m}_Z^2}{M_Z^2} \left(\sqrt{1 + 4 \frac{M_Z^2}{\tilde{m}_Z^2}} - 1 \right) \right]. \end{aligned} \quad (2.23)$$

Figure 1: The neutralino masses as a function of the photino mass.

The product of these neutralino masses yields the relation

$$\tilde{M}_h \tilde{M}_Z = M_Z^2. \quad (2.24)$$

As seen above, $\tilde{M}_Z \geq M_Z$ while $\tilde{M}_h \leq M_Z$. This places an upper bound phenomenological restriction on the photino mass, \tilde{M}_γ , since \tilde{M}_h cannot be too small. These masses are plotted in Figure 1 as a function of the photino mass.

Multiplying the chargino mass matrix by its Hermitian conjugate, $\tilde{\mu}_{ch}^2 = \tilde{M}_{ch}^\dagger \tilde{M}_{ch}$, the squared mass matrix is determined

$$\tilde{\mu}_{ch}^2 = \begin{pmatrix} \tilde{m}_W^2 + 2M_W^2 \sin^2 \beta & \sqrt{2}\tilde{m}_W M_W \cos \beta \\ \sqrt{2}\tilde{m}_W M_W \cos \beta & 2M_W^2 \cos^2 \beta \end{pmatrix}. \quad (2.25)$$

The charged fermion squared mass eigenvalues are found to be

$$\begin{aligned} \tilde{M}_{W^\pm}^2 &= M_W^2 \left[1 + \frac{1}{2} \frac{\tilde{m}_W^2}{M_W^2} \left(\sqrt{1 + 4 \frac{M_W^2}{\tilde{m}_W^2} + 4 \frac{M_W^4}{\tilde{m}_W^4} \cos^2 2\beta} + 1 \right) \right] \\ \tilde{M}_{h^\pm}^2 &= M_W^2 \left[1 - \frac{1}{2} \frac{\tilde{m}_W^2}{M_W^2} \left(\sqrt{1 + 4 \frac{M_W^2}{\tilde{m}_W^2} + 4 \frac{M_W^4}{\tilde{m}_W^4} \cos^2 2\beta} - 1 \right) \right]. \end{aligned} \quad (2.26)$$

Figure 2: The chargino masses as a function of the photino mass for various values of β .

The product of these chargino masses yields the relation

$$\tilde{M}_{W^\pm} \tilde{M}_{h^\pm} = M_W^2 \sin 2\beta. \quad (2.27)$$

Thus, one chargino mass is greater than M_W while the other is less than M_W . For small $\sin 2\beta$ (i.e., β close to 0 or $\frac{\pi}{2}$), the charged higgsino mass becomes unacceptably small. This results in phenomenological restrictions on the values of β . The chargino masses are plotted in Figure 2 as a function of the photino mass for various values of β .

The Goldstone scalar fields' effective potential is given by

$$\begin{aligned} V_G = & -\frac{1}{2}D_Y^2 - \frac{1}{2}\vec{D}_W \cdot \vec{D}_W - D^A J_A \\ & +(a+b)A_T^\dagger A_T + (a-b)A_B^\dagger A_B, \end{aligned} \quad (2.28)$$

where we have included the soft SUSY breaking terms a and b and have trivially eliminated the auxiliary fields, F^i , by means of their equations of

motion. Recall that the first component of the Goldstone field gauge current [31] is given by equation (B.35)

$$J_A = A_T^\dagger T_T^A A_T + A_B^\dagger T_B^A A_B. \quad (2.29)$$

From Appendix B, this becomes

$$\begin{aligned} V_G &= \frac{1}{2}g_2^2 \left(A_T^\dagger T^i A_T - A_B^\dagger T^i A_B \right)^2 + \frac{1}{8}g_1^2 \left(A_T^\dagger A_T - A_B^\dagger A_B \right)^2 \\ &\quad + (a+b)A_T^\dagger A_T + (a-b)A_B^\dagger A_B \\ &= \frac{(g_1^2 + g_2^2)}{8} \left(A_T^\dagger A_T - A_B^\dagger A_B \right)^2 + \frac{g_2^2}{2} \left(A_T^\dagger A_B \right)^2 \\ &\quad + (a+b)A_T^\dagger A_T + (a-b)A_B^\dagger A_B. \end{aligned} \quad (2.30)$$

Expanding about the vacuum values

$$\begin{aligned} \langle H_T \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_T \\ 0 \end{pmatrix} \\ \langle H_B \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_B \end{pmatrix}, \end{aligned} \quad (2.31)$$

or in terms of the doublet and SUSY component fields' vacuum values

$$\begin{aligned} \langle A_\sigma \rangle &= \sigma \\ \langle A^3 \rangle &= -i\eta, \end{aligned} \quad (2.32)$$

where the vacuum values σ and η are related to v_T and v_B by

$$\begin{aligned} \frac{1}{\sqrt{2}}v_T &= \sigma + \eta = \frac{1}{\sqrt{2}}v \cos \beta \\ \frac{1}{\sqrt{2}}v_B &= \sigma - \eta = \frac{1}{\sqrt{2}}v \sin \beta, \end{aligned} \quad (2.33)$$

the minimum of the effective potential is found to occur at

$$-\frac{1}{2}M_Z^2 = a + b \sec 2\beta, \quad (2.34)$$

which relates the soft SUSY breaking parameters a and b to the Z mass and the angle β . Note that $b = 0$ for $\beta = \frac{\pi}{4}$ ($v_T = v_B$), and is non-zero for unequal values of v_T and v_B , that is when $\langle A^3 \rangle = -i\eta \neq 0$.

The scalar field mass matrix decouples into a charged scalar field and a neutral scalar field matrix. In the complex charged electroweak basis, $A_-^\dagger = (A^{1\dagger} - iA^{2\dagger})$, $A_+ = (A^1 - iA^2)$, the Hermitian mass matrix is

$$M_{ch}^2 = (M_W^2 + 2a) \begin{pmatrix} \sin^2 \beta & -\frac{1}{2} \sin 2\beta \\ -\frac{1}{2} \sin 2\beta & \cos^2 \beta \end{pmatrix}. \quad (2.35)$$

This can be diagonalized by introducing the Goldstone boson fields

$$\begin{aligned} w_+ &\equiv (A_-^\dagger \cos \beta + A_+ \sin \beta) \\ w_- &\equiv (A_- \cos \beta + A_+^\dagger \sin \beta), \end{aligned} \quad (2.36)$$

which are massless (in the Stueckelberg gauge or have mass M_W in the Feynman- R_ξ gauge) and the charged Higgs fields (SUSY partners to the charged Goldstone bosons w_\pm)

$$\begin{aligned} h_+ &\equiv (A_+ \cos \beta - A_-^\dagger \sin \beta) \\ h_- &\equiv (A_+^\dagger \cos \beta - A_- \sin \beta), \end{aligned} \quad (2.37)$$

with mass squared

$$M_{h^\pm}^2 = M_W^2 + 2a. \quad (2.38)$$

Note that $M_{h^\pm}^2 \geq 0$ requires that $a \geq -\frac{1}{2}M_W^2$. Further, the phenomenological lower bound for M_{h^\pm} implies a lower bound for a . The charged Higgs mass is plotted in Figure 3 as a function of $\sqrt{|a|}$ for positive and negative values of a .

Figure 3: The charged Higgs mass as a function of the soft SUSY breaking parameter, $(\pm)|a|^{\frac{1}{2}}$, the $+$ denoting positive values of a and the $-$ denoting negative values of a .

These fields can be grouped to make manifest the rotational nature of their linear combinations

$$\begin{pmatrix} w_- \\ h_- \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} A_- \\ A_+^\dagger \end{pmatrix}, \quad (2.39)$$

and its Hermitian conjugate relation

$$(w_+ \quad h_+) = (A_-^\dagger \quad A_+) \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}. \quad (2.40)$$

The neutral scalar field mass matrix in the A_3, A_3^\dagger basis is

$$\frac{1}{2}M^2 = \frac{(M_Z^2 - 2a\sin^2 2\beta)}{1 + \sin 2\beta} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \quad (2.41)$$

Recall that we must rescale the neutral fields by a finite wavefunction renormalization factor. This follows from the form of the Kähler metric in the

Figure 4: The neutral Higgs mass as a function of the charged Higgs mass for various values of β .

vacuum

$$\begin{aligned}
g_{ij}|_{A_\sigma=\sigma, A^3=h} &= 2\delta_{ij} + \frac{2\eta^2}{\sigma^2}\delta_i^3\delta_j^3 \\
&= \begin{cases} 2\delta_{ij} & \text{for } i, j = 1, 2 \\ \frac{4}{1+\sin 2\beta} & \text{for } i = j = 3 \\ 0 & \text{otherwise} \end{cases} .
\end{aligned} \tag{2.42}$$

Introducing the renormalized neutral Goldstone boson field

$$z \equiv \sqrt{\frac{2}{1+\sin 2\beta}} (A^3 + A^{3\dagger}) \tag{2.43}$$

and the renormalized neutral Higgs field (SUSY partner to the z)

$$h \equiv i\sqrt{\frac{2}{1+\sin 2\beta}} (A^3 - A^{3\dagger}), \tag{2.44}$$

the matrix is diagonalized with z massless (in the Stueckelberg gauge or having mass M_Z in the Feynman- R_ξ gauge) and h having mass squared

$$M_h^2 = M_Z^2 + 2a\sin^2 2\beta. \tag{2.45}$$

Note that $M_h^2 \geq 0$ requires that $a \sin^2 2\beta \geq -\frac{1}{2}M_Z^2$, which is always satisfied for $a \geq -\frac{1}{2}M_W^2$. Using these masses, the minimum condition for the potential, equation (2.34), can be written as

$$-\frac{1}{2} \left(M_Z^2 - M_W^2 + M_{h^\pm}^2 \right) = b \sec 2\beta, \quad (2.46)$$

while the neutral Higgs mass squared, equation (2.45), is given by

$$M_h^2 = M_Z^2 + \left(M_{h^\pm}^2 - M_W^2 \right) \sin^2 2\beta. \quad (2.47)$$

Note that if $M_h \leq M_Z$, then $M_{h^\pm} \leq M_W$ and vice versa. Finally, using equation (2.27) for the chargino masses, the neutral Higgs mass equation (2.47) becomes

$$\left(M_h^2 - M_Z^2 \right) M_W^4 = \left(M_{h^\pm}^2 - M_W^2 \right) \tilde{M}_{W^\pm}^2 \tilde{M}_{h^\pm}^2. \quad (2.48)$$

The neutral Higgs mass is plotted in Figure 4 as a function of the charged Higgs mass for various values of β .

3 Longitudinal Vector Boson Scattering

The equivalence theorem provides a simple means to calculate the high energy scattering amplitudes for the longitudinal gauge bosons W_L^\pm, Z_L . They are equal to the same amplitudes with the longitudinal vector bosons replaced by their corresponding Goldstone fields, $W_L^\pm \rightarrow w_\pm, Z_L \rightarrow z$. Since the Goldstone bosons are derivatively coupled to each other, the dominant contributions to their high energy, $E \gg M_Z$, scattering amplitudes can be obtained from the kinetic energy terms in their Kähler lagrangian

$$\mathcal{L} = \partial_\lambda A^{i\dagger} \bar{g}_{ij}(\vec{A}) \partial^\lambda A^j, \quad (3.1)$$

with the metric

$$\begin{aligned} g_{ij} &= 2 \left\{ \delta_{ij} + \frac{\partial A_\sigma^\dagger}{\partial A^{i\dagger}} \frac{\partial A_\sigma}{\partial A^j} \right\} \\ &= 2 \left\{ \delta_{ij} + \frac{A^{i\dagger} A^j}{A_\sigma^\dagger A_\sigma} \right\} \\ &= 2 \left\{ \delta_{ij} + \frac{A^{i\dagger} A^j}{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^2} \sqrt{\frac{1}{2} v_T v_B - \vec{A}^2}} \right\}. \end{aligned} \quad (3.2)$$

In the tree approximation, the dominant contributions arise from direct quartic field terms and from one particle exchange graphs made of trilinear field terms. Hence, the metric must be expanded about the vacuum values, recall $\langle A^3 \rangle = -i\eta = \frac{-i}{\sqrt{8}}(v_T - v_B)$, through second order in the fields. Writing the complex fields A^i in terms of their shifted real components P^i, S^i as

$$A^i = P^i - iS^i - i\eta\delta_3^i, \quad (3.3)$$

we find through bilinear in fields

$$g_{ij} = \delta_{ij} + \frac{\eta^2}{\sigma^2} \delta_i^3 \delta_j^3 - \frac{2\eta^3}{\sigma^4} S^3 \delta_i^3 \delta_j^3 + \frac{i\eta}{\sigma^2} \left[\delta_i^3 (P^j - iS^j) - \delta_j^3 (P^i + iS^i) \right]$$

$$\begin{aligned}
& + \frac{1}{\sigma^2} \left[P^i P^j + S^i S^j + i \left(S^i P^j - P^i S^j \right) \right] \\
& - \frac{2i\eta^2}{\sigma^4} \left[\delta_i^3 \left(P^j - iS^j \right) - \delta_j^3 \left(P^i + iS^i \right) \right] S^3 \\
& + \frac{\eta^2}{\sigma^4} \delta_i^3 \delta_j^3 \left[P_1^2 + P_2^2 - S_1^2 - S_2^2 + \left(1 - \frac{2\eta^2}{\sigma^2} \right) P_3^2 - \left(1 - \frac{4\eta^2}{\sigma^2} \right) S_3^2 \right].
\end{aligned} \tag{3.4}$$

Recall $\eta = \frac{1}{\sqrt{8}}(v_T - v_B) = \frac{v}{\sqrt{8}}(\cos \beta - \sin \beta)$ and $\sigma = \frac{1}{\sqrt{8}}(v_T + v_B) = \frac{v}{\sqrt{8}}(\cos \beta + \sin \beta)$ so that

$$\begin{aligned}
8\eta^2 &= v^2 (1 - \sin 2\beta) \\
8\sigma^2 &= v^2 (1 + \sin 2\beta).
\end{aligned} \tag{3.5}$$

The charged Goldstone bosons, w_\pm , and the physical charged Higgs bosons, h_\pm , are given by linear combinations of the complex A^i fields as in equations (2.36) and (2.37). Hence, the charged component fields $A_\pm = P_\pm - iS_\pm$, that is

$$\begin{aligned}
P_- &= \frac{1}{2} (A_- + A_+^\dagger), \quad S_- = \frac{i}{2} (A_- - A_+^\dagger) \\
P_+ &= \frac{1}{2} (A_-^\dagger + A_+), \quad S_+ = \frac{i}{2} (A_+ - A_-^\dagger),
\end{aligned} \tag{3.6}$$

are related to the mass eigenstate fields by

$$\begin{aligned}
P_\pm &= \frac{1}{2} (\cos \beta + \sin \beta) w_\pm + \frac{1}{2} (\cos \beta - \sin \beta) h_\pm \\
S_\pm &= \mp \frac{i}{2} (\cos \beta - \sin \beta) w_\pm \pm \frac{i}{2} (\cos \beta + \sin \beta) h_\pm.
\end{aligned} \tag{3.7}$$

Moreover, in order to cast the kinetic terms of the neutral Goldstone boson and Higgs boson in their conventional form a finite wavefunction renormalization is required:

$$P^3 = \frac{1}{2} \sqrt{\frac{1 + \sin 2\beta}{2}} z$$

$$S^3 = \frac{1}{2} \sqrt{\frac{1 + \sin 2\beta}{2}} h. \quad (3.8)$$

After some algebra, the Feynman rules can be gleaned from the Lagrangian. The various scattering amplitudes are calculated in the Stueckelberg gauge in which the Goldstone bosons are massless

$$\begin{aligned} T(w^+ w^- \rightarrow w^+ w^-) &= \frac{i}{v^2} \frac{2 \sin^2 2\beta}{1 + \sin 2\beta} \left[s + t - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{s^2}{s - M_h^2} + \frac{t^2}{t - M_h^2} \right\} \right], \\ T(z z \rightarrow w^+ w^-) &= \frac{i}{v^2} \frac{2 \sin^2 2\beta}{1 + \sin 2\beta} \left[s - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{s^2}{s - M_h^2} \right\} \right], \\ T(z z \rightarrow z z) &= \frac{i}{v^2} \frac{2 \sin^2 2\beta}{1 + \sin 2\beta} \left[s + t + u - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{s^2}{s - M_h^2} + \frac{t^2}{t - M_h^2} + \frac{u^2}{u - M_h^2} \right\} \right]. \end{aligned} \quad (3.9)$$

Besides the enhancement of the pure Goldstone boson scattering processes at high energy, SUSY implies a similar enhancement for the Goldstone boson to Higgs boson production processes. The tree approximation leading contributions also follow from the second order expansion of the metric, equation (3.4). After some algebra, the Higgs production amplitudes are secured

$$\begin{aligned} T(z z \rightarrow h^0 h^0) &= -\frac{i}{v^2} \frac{2 \sin^2 2\beta}{1 + \sin 2\beta} \left[2s - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ s \frac{s - 2M_h^2}{s - M_h^2} - \frac{(t - M_h^2)^2}{t} - \frac{(u - M_h^2)^2}{u} + 6 \frac{M_h^4}{\sin 2\beta} \right\} \right], \\ T(z z \rightarrow h^+ h^-) &= -\frac{i}{v^2} \frac{2 \sin^2 2\beta}{1 + \sin 2\beta} \left[s - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{s^2}{s - M_h^2} \right\} \right], \end{aligned}$$

$$\begin{aligned}
T(w^+w^- \rightarrow h^0h^0) &= -\frac{i}{v^2} \frac{2 \sin^2 2\beta}{1 + \sin 2\beta} \left[s - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ s \frac{s - 2M_h^2}{s - M_h^2} \right\} \right] \\
&\quad - \frac{i}{v^2} \frac{2 \cos^2 2\beta}{1 + \sin 2\beta} \left[\cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{M_h^4}{u - M_{h^\pm}^2} + \frac{M_h^4}{t - M_{h^\pm}^2} \right\} \right], \\
T(w^+w^- \rightarrow h^+h^-) &= -\frac{i}{v^2} \frac{2 \sin^2 2\beta}{1 + \sin 2\beta} \left[s - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{s^2}{s - M_h^2} \right\} \right] \\
&\quad + \frac{i}{v^2} \frac{2}{1 + \sin 2\beta} \left[t - \cos^2\left(\frac{\pi}{4} + \beta\right) \{t\} \right] \\
&\quad + \frac{i}{v^2} \frac{2 \cos^2 2\beta}{1 + \sin 2\beta} \left[t - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{t^2}{t - M_h^2} \right\} \right] \\
T(w^\pm w^\pm \rightarrow h^\pm h^\pm) &= +\frac{4i}{v^2} \frac{\cos^2 2\beta}{1 + \sin 2\beta} \left[t + u - \cos^2\left(\frac{\pi}{4} + \beta\right) \left\{ \frac{t^2}{t - m_0^2} + \frac{u^2}{u - m_0^2} \right\} \right] \\
&\quad + \frac{2is}{v^2} \\
T(zw^\pm \rightarrow hh^\pm) &= -\frac{t}{v^2} \sin 2\beta - \frac{m_0^2}{v^2} \frac{1 - \sin 2\beta}{1 + \sin 2\beta} + \frac{2m_0^2}{v^2} \frac{\sin 2\beta}{1 + \sin 2\beta} \cos^2\left(\frac{\pi}{4} + \beta\right).
\end{aligned} \tag{3.10}$$

For energies above the charged and neutral Higgs masses, as well as M_Z , equation (3.9) reduces to the scattering amplitudes for strongly interacting longitudinal gauge bosons

$$\begin{aligned}
T(W_L^+W_L^- \rightarrow W_L^+W_L^-) &\approx -\frac{iu}{v^2} \sin^2 2\beta, \\
T(Z_L Z_L \rightarrow W_L^+W_L^-) &\approx +\frac{is}{v^2} \sin^2 2\beta, \\
T(Z_L Z_L \rightarrow Z_L Z_L) &\approx 0.
\end{aligned} \tag{3.11}$$

Furthermore, the enhanced Higgs boson production amplitudes from longitudinal vector boson scattering, equation (3.10), become

$$T(Z_L Z_L \rightarrow h^0 h^0) \approx -\frac{2is}{v^2} \sin^2 2\beta,$$

$$\begin{aligned}
T(Z_L Z_L \rightarrow h^+ h^-) &\approx -\frac{is}{v^2} \sin^2 2\beta, \\
T(W_L^+ W_L^- \rightarrow h^0 h^0) &\approx -\frac{is}{v^2} \sin^2 2\beta, \\
T(W_L^+ W_L^- \rightarrow h^+ h^-) &\approx -\frac{is}{v^2} \sin^2 2\beta + \frac{it}{v^2} (1 + \cos^2 2\beta) \\
T(W_L^\pm W_L^\pm \rightarrow h^\pm h^\pm) &\approx +\frac{2is}{v^2} \sin^2 2\beta \\
T(Z_L W_L^\pm \rightarrow h h^\pm) &\approx -\frac{t}{v^2} \sin 2\beta.
\end{aligned} \tag{3.12}$$

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Appendix A The Heavy Higgs Limit

The SUSY non-linear realization of the electroweak symmetry breakdown, $SU(2) \times U(1) \rightarrow U(1)$, can be obtained directly by considering the heavy Higgs limit of the (M+1)SSM. The Higgs sector is comprised of the two $SU(2)$ doublet chiral superfields, with opposite weak hypercharge, H_B and H_T , and an additional singlet chiral superfield, N . The supersymmetric action for these fields is given by

$$\begin{aligned} \Gamma &= \int dV \left[\bar{H}_B H_B + \bar{H}_T H_T + \bar{N} N \right] \\ &- \int dS \left[\xi N + \frac{1}{2} m N^2 + \frac{\lambda}{3!} N^3 + \mu H_T \epsilon H_B - g N H_T \epsilon H_B \right] + h.c. \end{aligned} \quad (\text{A.1})$$

The superfield Euler-Lagrange equations are given by field derivatives of the superpotential so that

$$\begin{aligned} \bar{D} \bar{D} \bar{N} &= \xi + m N + \frac{\lambda}{2} N^2 - g H_T \epsilon H_B, \\ \bar{D} \bar{D} \bar{H}_B &= (\mu - g N) H_T \epsilon, \\ \bar{D} \bar{D} \bar{H}_T &= (\mu - g N) \epsilon H_B. \end{aligned} \quad (\text{A.2})$$

In the strong coupling limit, the superpotential derivative terms on the right hand sides above must vanish for a non-vanishing effective action. Hence we find the constraints

$$\begin{aligned} N &= \frac{m u}{g} \equiv n < \infty, \\ H_T \epsilon H_B &= \frac{\xi}{g} + \frac{m \mu}{g^2} + \frac{\lambda}{2} \frac{\mu^2}{g g^2} \equiv \frac{1}{4} v^2 \sin 2\beta < \infty. \end{aligned} \quad (\text{A.3})$$

The heavy singlet field becomes stationary at its vacuum value in the strong coupling limit $\mu, g \rightarrow \infty$ with $n = \frac{m u}{g}$ fixed. Likewise the chiral composite

field $H_T \epsilon H_B$ becomes stationary at its vacuum value as $g \rightarrow \infty$ with at least one or all $\frac{\xi}{g}$, $\frac{m}{g}$, $\frac{\lambda}{g}$ fixed. This results in the heavy neutral scalar field, the pseudoscalar field and their heavy neutral fermion partner field in H_B and H_T to become heavy. This can be made clearer by introducing the σ -model notation of equation (2.1). Hence $H_T \epsilon H_B = \sigma^2 + \vec{\pi}^2 = \frac{1}{2} v_T v_B$, allowing the σ superfield to be eliminated as in equation (2.2), $\sigma = \sqrt{\frac{1}{2} v_B v_T - \vec{\pi}^2}$, from the action, inducing non-linear π^i self interactions from the σ -kinetic energy terms, as is familiar.

Recalling the component field expansion for σ and $\vec{\pi}$,

$$\begin{aligned}\sigma &= e^{-i\theta\phi\bar{\theta}} \left[A_\sigma - i\sqrt{2}\theta\psi_\sigma + \theta^2 F_\sigma \right], \\ \pi^i &= e^{-i\theta\phi\bar{\theta}} \left[A^i - i\sqrt{2}\theta\psi^i + \theta^2 F^i \right].\end{aligned}\tag{A.4}$$

The constraint equation (A.3) can be expanded to yield the component field constraints

$$\begin{aligned}A_\sigma^2 + \vec{A}^2 &= \frac{1}{2} v_T v_B, \\ A_\sigma \psi_\sigma + \vec{A} \cdot \vec{\psi} &= 0, \\ A_\sigma F_\sigma + \vec{A} \cdot \vec{F} - \frac{1}{2} \psi_\sigma^2 - \frac{1}{2} \vec{\psi} \cdot \vec{\psi} &= 0.\end{aligned}\tag{A.5}$$

The heavy Higgs limit can be further clarified by considering the strong coupling limit of the component field action. In particular the mass matrix will yield a heavy mass for the singlet multiplet as well as for the σ -multiplet, that is the heavier neutral scalar, the neutral pseudoscalar and their neutral fermion partner field combinations of the H_B and H_T doublets. The linear σ -model action is given by

$$\Gamma = \int d^4x \left[\partial_\lambda A_B^\dagger \partial^\lambda A_B + \partial_\lambda A_T^\dagger \partial^\lambda A_T + \partial_\lambda A_N^\dagger \partial^\lambda A_N + \right.$$

$$\begin{aligned}
& + \frac{i}{2} \psi_B \overleftrightarrow{\not{\partial}} \bar{\psi}_B + \frac{i}{2} \psi_T \overleftrightarrow{\not{\partial}} \bar{\psi}_T + \frac{i}{2} \psi_N \overleftrightarrow{\not{\partial}} \bar{\psi}_N \\
& + F_B^\dagger F_B + F_T^\dagger F_T + F_N^\dagger F_N \\
& + \left\{ F_N \left[4g A_T \epsilon A_B - 4\xi - 4m A_N - 2\lambda A_N^2 \right] \right. \\
& + 4(g A_N - \mu) [F_T \epsilon A_B + A_T \epsilon F_B - \psi_T \epsilon \psi_B] \\
& \left. + 2(m + \lambda A_N) \psi_N^2 - 4g \psi_N [A_T \epsilon \psi_B + \psi_T \epsilon A_B] + h.c. \right\}. \quad (\text{A.6})
\end{aligned}$$

Note that the strong coupling limit $g, \xi \rightarrow \infty$ with $\frac{\xi}{g}$ fixed yields the doublet fields' constraint equations (A.3)

$$\begin{aligned}
A_T \epsilon A_B &= \frac{\xi}{g} \equiv \frac{1}{2} v_T v_B \\
A_T \epsilon \psi_B + \psi_T \epsilon A_B &= 0 \\
F_T \epsilon A_B + A_T \epsilon F_B - \psi_T \epsilon \psi_B &= 0
\end{aligned} \quad (\text{A.7})$$

while $\lambda, m \rightarrow \infty$ and $\frac{2m}{\lambda}$ fixed yields the decoupled stationary massive singlet multiplet $A_N = \frac{2m}{\lambda} \equiv n, \psi_N = 0, F_N = 0$. The resulting SUSY non-linear σ -model lagrangian is given by the kinetic energy terms of the constrained, equation (A.7), doublet fields

$$\begin{aligned}
\mathcal{L} &= \partial_\lambda A_B^\dagger \partial^\lambda A_B + \partial_\lambda A_T^\dagger \partial^\lambda A_T + \frac{i}{2} \psi_B \overleftrightarrow{\not{\partial}} \bar{\psi}_B + \frac{i}{2} \psi_T \overleftrightarrow{\not{\partial}} \bar{\psi}_T \\
&+ F_B^\dagger F_B + F_T^\dagger F_T.
\end{aligned} \quad (\text{A.8})$$

Appendix B The Kähler Potential

The heavy Higgs limit of the (M+1)SSM will give rise to the simplest form of the lowest order SUSY non-linear sigma model action. The (M+1)SSM action can be written as

$$\Gamma_{(M+1)\text{SSM}} = \Gamma_G + \Gamma_M + \Gamma_{YM} + \Gamma_{FI} \quad (\text{B.1})$$

above Γ_{YM} , is the usual electroweak gauge field kinetic energy terms and soft *SUSY* breaking gaugino mass terms, along with the supersymmetric gauge fixing and Fadeev-Popov pieces. Γ_{FI} is a possible $U(1)$ hypercharge Fayet-Illiopoulous term. Γ_M is the gauge invariant supersymmetric quark and lepton superfield kinetic energy terms as well as their Yukawa-interactions along with their associated soft SUSY breaking terms. Likewise Γ_G are the Higgs doublet H_B , H_T and singlet N gauge invariant supersymmetric kinetic energy terms as well as the H_B , H_T , N superpotential terms and all the related soft SUSY breaking terms. Hence the Higgs field action terms are the gauge invariant version of those given in equation (A.1)

$$\begin{aligned} \Gamma_G = & \int dV \left[Z_B \bar{H}_B e^{-2V_{H_B}} \bar{H}_B + Z_T \bar{H}_T e^{-2V_{H_T}} \bar{H}_T + Z_N \bar{N} N \right] \\ & + \left\{ \int dS \left[\xi N + \frac{1}{2} m N^2 + \frac{1}{3!} \lambda N^3 + \mu H_T \epsilon H_B - g N H_T \epsilon H_B \right] + h.c. \right\}. \end{aligned} \quad (\text{B.2})$$

The residual supergravity soft-SUSY breaking terms are included as θ , $\bar{\theta}$ dependent wavefunction renormalization factors, $Z_\phi = (1 + m_\phi^2 \theta^2 \bar{\theta}^2)$, and coupling constants, $g = g + a_g \theta^2$, $\mu = \mu + a_\mu \theta^2$, etc., mimicking the constant graviton and gravitino coupling effects [30][32]. The matter field terms can be similarly written. They do not involve the singlet field and are as in the

MSSM:

$$\begin{aligned}
\Gamma_M = & \int dV \left[Z_Q \bar{Q} e^{-2V_Q} Q + Z_{T^C} \bar{T}^C e^{-2V_{T^C}} T^C + Z_{B^C} \bar{B}^C e^{-2V_{B^C}} B^C \right. \\
& + \left. Z_{E^C} \bar{E}^C e^{-2V_{E^C}} E^C + Z_L \bar{L} e^{-2V_L} L \right] \\
& + \left\{ \int dS \left[g_E^{mn} E_m^C L_n^a H_B^a + g_B^{mn} B_m^C Q_n^a H_B^a + g_T^{mn} T_m^C Q_n^a H_T^a \right] + h.c. \right\}.
\end{aligned} \tag{B.3}$$

where as before the wavefunction renormalization factors include the soft SUSY breaking masses, $Z_\phi = (1 + m_\phi^2 \theta^2 \bar{\theta}^2)$, and the coupling constants have the form $g = g + a_g \theta^2$. The vector gauge fields are defined as

$$\begin{aligned}
V_Q &= -g_3 \vec{G} \cdot \frac{\vec{\lambda}}{2} - g_2 \vec{W} \cdot \frac{\vec{\sigma}}{2} - \frac{1}{6} g_1 Y \\
V_{T^C} &= +g_3 \vec{G} \cdot \frac{\vec{\lambda}}{2} + \frac{2}{3} g_1 Y \\
V_{B^C} &= +g_3 \vec{G} \cdot \frac{\vec{\lambda}}{2} - \frac{1}{3} g_1 Y \\
V_{E^C} &= -g_1 Y \\
V_L &= -g_2 \vec{W} \cdot \frac{\vec{\sigma}}{2} + \frac{1}{2} g_1 Y \\
V_B &= +\frac{1}{2} g_1 Y + g_2 \vec{W} \cdot \vec{T} \\
V_T &= -\frac{1}{2} g_1 Y + g_2 \vec{W} \cdot \vec{T},
\end{aligned} \tag{B.4}$$

with the $U(1)$ generators represented by the identity, the $SU(2)$ generators given by the Pauli matrices, $T^i = \frac{\sigma^i}{2}$, $i = 1, 2, 3$, and the $SU(3)$ generators represented by the Gell-Mann matrices, $L^a = \frac{\lambda^a}{2}$, $a = 1, \dots, 8$.

The gauge invariant Yang-Mills terms have the structure

$$\Gamma_{YM}^{inv} = \int dS \frac{Z_W}{4g_2^2} \text{Tr} [W^\alpha W_\alpha] + \int dS \frac{Z_Y}{4g_1^2} Y^\alpha Y_\alpha + h.c., \tag{B.5}$$

with the field strenth spinors given as

$$W^\alpha = \bar{D}\bar{D} \left[e^{-2g_2\vec{W}\cdot\frac{\vec{\sigma}}{2}} D^\alpha e^{2g_2\vec{W}\cdot\frac{\vec{\sigma}}{2}} \right], \quad (\text{B.6})$$

$$Y^\alpha = \bar{D}\bar{D} \left[e^{-2g_1Y} D^\alpha e^{2g_1Y} \right] = 2g_1\bar{D}\bar{D}D^\alpha Y. \quad (\text{B.7})$$

The gaugino mass terms result from the soft SUSY breaking terms contained in the wavefunction normalization factors

$$\begin{aligned} Z_W &= \left(1 + \frac{1}{2}\tilde{m}_W\theta\theta \right), \\ Z_Y &= \left(1 + \frac{1}{2}\tilde{m}_Y\theta\theta \right). \end{aligned} \quad (\text{B.8})$$

The Fayet-Iliopoulous hypercharge term is simply

$$\Gamma_{FI} = \kappa \int dV Y. \quad (\text{B.9})$$

Since, after eliminating the auxiliary hypercharge field, D_Y , this is equivalent to the soft SUSY breaking b -terms in the Higgs action. Hence, we set $\kappa = 0$, incorporating the possibility of a Fayet-Iliopoulous term in the b -breaking terms. For a more detailed display of the MSSM and (M+1)SSM notation conventions used here see [33][9].

The strong coupling limit for the singlet and doublet Higgs fields results in the $\Gamma_{(M+1)\text{SSM}}$ becoming a constrained field action. Indeed, we have that, in analogy to the discussion in Appendix A, Γ_G becomes simply the kinetic energy terms for the constained doublet fields,

$$\Gamma_G = \int dV \left[Z_{H_B} \bar{H}_B e^{-2V_{H_B}} H_B + Z_{H_T} \bar{H}_T e^{-2V_{H_T}} H_T \right] \quad (\text{B.10})$$

where $H_T \epsilon H_B = \frac{1}{2} v_T v_B$ and we can write the wavefunction renormalization factors as $Z_{H_B} = 1 + (a - b)\theta^2 \bar{\theta}^2$, $Z_{H_T} = 1 + (a + b)\theta^2 \bar{\theta}^2$. The superpotential

with the soft breaking terms and the singlet field kinetic energy terms are a constant or zero. The remaining action is as in the MSSM except for the Yukawa interaction terms. Since they involve the constrained field, new interactions with the Goldstone superfields will occur as the constrained fields are eliminated in favor of the Goldstone multiplet fields. The form of the (s)quark and (s)lepton mass terms will however be the same as in the MSSM since the form of the vacuum values of H_B and H_T are unchanged.

Generally the Goldstone multiplet action can be written in terms of a gauge invariant extension of the Kähler potential $K = K(\vec{\pi}, \vec{\bar{\pi}})$ (see reference [31] for notation conventions). This can be accomplished most directly by simply implementing the constraints

$$\begin{aligned}\sigma &= \sqrt{\frac{1}{2}v_T v_B - \vec{\pi}^2}, \\ \bar{\sigma} &= \sqrt{\frac{1}{2}v_T v_B - \vec{\bar{\pi}}^2}.\end{aligned}\tag{B.11}$$

Hence the gauge invariant extension of the super Kähler potential including soft SUSY breaking terms is found to be

$$K = Z_{H_B} \bar{H}_B e^{-2V_{H_B}} H_B + Z_{H_T} \bar{H}_T e^{-2V_{H_T}} H_T,\tag{B.12}$$

with

$$\begin{aligned}H_B &= \begin{bmatrix} i\pi^+ \\ \sqrt{\frac{1}{2}v_T v_B - \vec{\pi}^2} - i\pi^0 \end{bmatrix}, \\ H_T &= \begin{bmatrix} \sqrt{\frac{1}{2}v_T v_B - \vec{\pi}^2} + i\pi^0 \\ i\pi^- \end{bmatrix}.\end{aligned}\tag{B.13}$$

The Kähler structure can be further manifested by expanding the superfields in terms of their component fields as given in equations (2.13). The full

action, Γ_K , (in the Wess-Zumino gauge) including the Yang-Mills auxiliary field terms and the soft SUSY breaking gaugino mass terms becomes (see equations (2.9-2.12, 2.14-2.16)) $\Gamma_K = \int d^4x \mathcal{L}_K$, with \mathcal{L}_K given by

$$\mathcal{L}_K = \mathcal{L}_G + \mathcal{L}_{YMD} + \mathcal{L}_{YM\cancel{\$}} \quad (\text{B.14})$$

where $\Gamma_G = \int dx \mathcal{L}_G$ and

$$\mathcal{L}_G = \mathcal{L}_{GS} + \mathcal{L}_{G\cancel{\$}}. \quad (\text{B.15})$$

Expanding in terms of component fields, the individual component field lagrangians are found to be

$$\begin{aligned} \mathcal{L}_{GS} &= (D_\lambda A_T)^\dagger (D^\lambda A_T) + i\bar{\psi}_T \bar{D} \psi_T + F_T^\dagger F_T - A_T^\dagger D_T A_T \\ &\quad + \sqrt{2} [\bar{\psi}_T \lambda_T A_T + A_T^\dagger \lambda_T \psi_T] + (T \longrightarrow B) \\ \mathcal{L}_{G\cancel{\$}} &= (a+b) A_T^\dagger A_T + (a-b) A_B^\dagger A_B \\ \mathcal{L}_{YMD} &= -\frac{1}{2} \vec{D}_W \cdot \vec{D}_W - \frac{1}{2} D_Y^2 \\ \mathcal{L}_{YM\cancel{\$}} &= \frac{1}{2} \tilde{m}_W \vec{\lambda}_W \cdot \vec{\lambda}_W + \frac{1}{2} \tilde{m}_Y \lambda_Y^2 + h.c. \\ &= \frac{1}{2} \begin{pmatrix} \lambda_\gamma & \lambda_Z \end{pmatrix} \begin{pmatrix} \tilde{m}_\gamma & \tilde{m}_{\gamma Z} \\ \tilde{m}_{\gamma Z} & \tilde{m}_Z \end{pmatrix} \begin{pmatrix} \lambda_\gamma \\ \lambda_Z \end{pmatrix} + \tilde{m}_W \lambda_+ \lambda_- + h.c., \end{aligned} \quad (\text{B.16})$$

where the covariant derivatives follow from equation (B.4)

$$\begin{aligned} D^\mu A_T &= \partial^\mu A_T + iV_T^\mu A_T \\ D^\mu \psi_T &= \partial^\mu \psi_T + iV_T^\mu \psi_T, \end{aligned} \quad (\text{B.17})$$

and analogously for A_B and ψ_B .

The constraints can be solved to yield the doublet component fields dependence on \vec{A} , $\vec{\psi}$, \vec{F} . This has the generic structure (recall each chiral superfield, constrained or not, has the component field structure as defined in

equation (2.13, A.4))

$$\begin{aligned}
A_T^a &= H_T^a|_{\theta\bar{\theta}=0} \equiv H_T^a(\vec{A}) \\
\psi_T^a &= \frac{\partial}{\partial\theta} H_T^a|_{\theta\bar{\theta}=0} = \frac{\partial H_T^a(\vec{A})}{\partial A^i} \psi^i \\
F_T^a &= -\frac{1}{4} \frac{\partial^2}{\partial\theta^2} H_T^a|_{\theta\bar{\theta}=0} = \left\{ F^i \frac{\partial}{\partial A^i} + \frac{1}{2} \psi^i \psi^j \frac{\partial^2}{\partial A^i \partial A^j} \right\} H_T^a(\vec{A}), \quad (\text{B.18})
\end{aligned}$$

and similarly for H_B and its components. In particular, we find that the constrained components of σ are given by

$$\begin{aligned}
A_\sigma &= \sqrt{\frac{1}{2} v_T v_B - \vec{A}^2} \\
\psi_\sigma &= -\frac{1}{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^2}} \vec{A} \cdot \vec{\psi} \\
F_\sigma &= -\frac{1}{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^2}} \left[\vec{A} \cdot \vec{F} - \frac{1}{2} \vec{\psi} \cdot \vec{\psi} - \frac{1}{2} \frac{(\vec{A} \cdot \vec{\psi})^2}{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^2}} \right]. \quad (\text{B.19})
\end{aligned}$$

In addition, the $SU(2) \times U(1)$ gauge transformations of the doublet superfields will define the Killing vector superfields, $A_A^i(\vec{\pi})$, in terms of the Goldstone superfields, π^i , (in this notation we label $\Lambda_Y = \Lambda_4$, so that the index A runs over all the generator labels 1, 2, 3, 4 of $SU(2) \times U(1)$ while index i runs over the broken generator labels 1, 2, 3 of $SU(2) \times U(1)/U(1)$ [31])

$$\begin{aligned}
\delta H_T &= \left(-\frac{i}{2} \Lambda_Y + i \vec{\Lambda} \cdot \vec{T} \right) H_T = i \Lambda^A T_T^A H_T \\
&\equiv \frac{\partial H_T}{\partial \pi^i} \delta \pi^i = \frac{\partial H_T}{\partial \pi^i} \Lambda^A A_A^i(\vec{\pi}) \\
\delta H_B &= \left(+\frac{i}{2} \Lambda_Y + i \vec{\Lambda} \cdot \vec{T} \right) H_B = i \Lambda^A T_B^A H_B \\
&\equiv \frac{\partial H_B}{\partial \pi^i} \delta \pi^i = \frac{\partial H_B}{\partial \pi^i} \Lambda^A A_A^i(\vec{\pi}). \quad (\text{B.20})
\end{aligned}$$

Expanding in powers of $\theta, \bar{\theta}$, the component field transformation equations

are secured

$$\begin{aligned}
iT_T^A A_T &= \frac{\partial H_T(\vec{A})}{\partial A^i} A_A^i(\vec{A}) \\
iT_T^A \psi_T &= \left(\frac{\partial H_T(\vec{A})}{\partial A^i} \frac{\partial A_A^i(\vec{A})}{\partial A^j} + \frac{\partial^2 H_T(\vec{A})}{\partial A^j \partial A^i} A_A^i(\vec{A}) \right) \psi^j \\
iT_T^A F_T &= \left\{ F^j \frac{\partial}{\partial A^j} + \frac{1}{2} \psi^j \psi^k \frac{\partial^2}{\partial A^j \partial A^k} \right\} \left[\frac{\partial H_T(\vec{A})}{\partial A^i} A_A^i(\vec{A}) \right], \quad (\text{B.21})
\end{aligned}$$

and likewise for the components of H_B . The specific form of the Killing vectors, A_A^i , will be discussed in Appendix C.

These relations now allow the component lagrangian \mathcal{L}_K to be put into its manifest Kähler form [34]. For example, the scalar field kinetic energy terms become

$$\begin{aligned}
\partial_\lambda A_T^\dagger \partial^\lambda A_T + \partial_\lambda A_B^\dagger \partial^\lambda A_B &= \partial_\lambda A^{\dagger i} \left[\frac{\partial \bar{H}_T^a}{\partial A^{\dagger i}} \frac{\partial H_T^a}{\partial A^j} + \frac{\partial \bar{H}_B^a}{\partial A^{\dagger i}} \frac{\partial H_B^a}{\partial A^j} \right] \partial^\lambda A^j \\
&= \partial_\lambda A^{\dagger i} g_{ij} \partial^\lambda A^j, \quad (\text{B.22})
\end{aligned}$$

where the Kähler manifold metric, g_{ij} , is given, as above, by the $\theta \bar{\theta}$ independent component of the superfield Kähler metric

$$g_{ij}(\vec{\pi}, \vec{\pi}) = \frac{\partial^2}{\partial \bar{\pi}^i \partial \pi^j} K(\vec{\pi}, \vec{\pi}). \quad (\text{B.23})$$

The simple super Kähler potential (ignoring the SUSY breaking terms and the gauge couplings) is $K(\vec{\pi}, \vec{\pi}) = \bar{H}_T H_T + \bar{H}_B H_B = 2(\bar{\sigma} \sigma + \vec{\pi} \cdot \vec{\pi})$, hence we find the component metric

$$\begin{aligned}
g_{ij} &= 2 \left(\delta_{ij} + \frac{\partial A_\sigma^\dagger}{\partial A^{\dagger i}} \frac{\partial A_\sigma}{\partial A^j} \right) \\
&= 2 \left(\delta_{ij} + \frac{A^{\dagger i} A^j}{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^2} \sqrt{\frac{1}{2} v_T v_B - \vec{A}^2}} \right). \quad (\text{B.24})
\end{aligned}$$

The gauge invariant scalar field kinetic energy terms in equation (B.16) yield those in terms of the complex Goldstone boson fields

$$\begin{aligned}
\mathcal{L} &= \partial_\lambda A_T^\dagger \partial^\lambda A_T - i A_T^\dagger V_T^\lambda \partial_\lambda A_T + i \partial_\lambda A_T^\dagger V_T^\lambda A_T \\
&\quad + A_T^\dagger V_{T\lambda} V_T^\lambda A_T + (T \rightarrow B) \\
&= \left(D^\lambda A \right)^{\dagger i} g_{\bar{i}j} (D_\lambda A)^j,
\end{aligned} \tag{B.25}$$

with the covariant derivative given by

$$(D_\mu A)^i \equiv \partial_\mu A^i + V_\mu^A A_A^i. \tag{B.26}$$

The gauge fields have been combined in the notation V_μ^A with (we choose $T_T^4 = -\frac{1}{2}$ and $T_B^4 = +\frac{1}{2}$ here)

$$\begin{aligned}
V_T^\mu &= -\frac{1}{2} g_1 Y^\mu + g_2 \vec{W}^\mu \cdot \vec{T} \\
&\equiv V^{A\mu} T_T^A \\
V_B^\mu &= +\frac{1}{2} g_1 Y^\mu + g_2 \vec{W}^\mu \cdot \vec{T} \\
&\equiv V^{A\mu} T_B^A,
\end{aligned} \tag{B.27}$$

so that the generalized vector field becomes

$$V_\mu^A = \begin{cases} g_2 W_\mu^i & \text{for } A = i = 1, 2, 3 \\ g_1 Y_\mu & \text{for } A = 4 \end{cases} \tag{B.28}$$

Analogously, each term in the constrained field lagrangian can be expressed in terms of the component fields. The lagrangian \mathcal{L}_K becomes

$$\begin{aligned}
\mathcal{L}_K &= \left(D^\lambda A \right)^{\dagger i} g_{\bar{i}j} (D_\lambda A)^j + \frac{i}{2} \bar{\psi}^i g_{\bar{i}j} (\overline{D} \psi)^j \\
&\quad - \frac{i}{2} (\overline{D^\mu \psi})^i g_{\bar{i}j} \bar{\sigma}_\mu \psi^j + D^A J_A \\
&\quad + \sqrt{2} \left(\bar{\psi}^i g_{\bar{i}j} A_A^j \bar{\lambda}^A + \lambda^A \bar{A}_A^i g_{\bar{i}j} \psi^j \right) + \frac{1}{4} R_{i\bar{k}j\bar{l}} (\bar{\psi}^k \bar{\psi}^l) (\psi^i \psi^j) \\
&\quad + \left(\bar{F}^i + \frac{1}{2} \bar{\Gamma}_{\bar{m}\bar{n}}^i (\bar{\psi}^m \bar{\psi}^n) \right) g_{\bar{i}j} \left(F^j + \frac{1}{2} \Gamma_{rs}^j (\psi^r \psi^s) \right).
\end{aligned} \tag{B.29}$$

The fermion covariant derivative, $(D_\mu \psi)^i$, is defined to be

$$(D_\mu \psi)^i = \partial_\mu \psi^i + V_\mu^A \frac{\partial A_A^i}{\partial A^j} \psi^j + \Gamma_{jk}^i (D_\mu A)^j \psi^j. \quad (\text{B.30})$$

The non-zero connection Γ_{jk}^i is defined by

$$\begin{aligned} \Gamma_{\bar{i}jk} &= \left[\frac{\partial \bar{H}_T^a(\vec{A}^\dagger)}{\partial A^{\dagger i}} \frac{\partial^2 H_T^a(\vec{A})}{\partial A^k \partial A^j} + \frac{\partial \bar{H}_B^a(\vec{A}^\dagger)}{\partial A^{\dagger i}} \frac{\partial^2 H_B^a(\vec{A})}{\partial A^k \partial A^j} \right] \\ &= \frac{\partial g_{ik}}{\partial A^j} = \frac{\partial g_{ij}}{\partial A^k} = \Gamma_{ikj}, \end{aligned} \quad (\text{B.31})$$

with $\Gamma_{jk}^i = g^{\bar{l}i} \Gamma_{\bar{l}jk}$, and correspondingly for the complex conjugate connection $\bar{\Gamma}_{\bar{j}\bar{k}}^{\bar{i}} = g^{\bar{l}i} \frac{\partial g_{\bar{k}\bar{l}}}{\partial A^{\dagger j}}$. The Riemann curvature tensor, $R_{i\bar{j}k\bar{l}}$, for the Kähler manifold is defined by $R_{i\bar{j}k\bar{l}} = g_{\bar{l}m} R_{i\bar{j}k}^m$ with

$$R_{i\bar{j}k}^m = \frac{\partial}{\partial A^{\dagger j}} \Gamma_{ik}^m. \quad (\text{B.32})$$

Hence we find that

$$R_{i\bar{j}k\bar{l}} = \frac{\partial}{\partial A^{\dagger j}} \Gamma_{\bar{l}ik} - \bar{\Gamma}_{\bar{j}l}^{\bar{n}} \Gamma_{\bar{n}ik}. \quad (\text{B.33})$$

Finally the Killing potentials, J_A , are given by the $\theta, \bar{\theta}$ independent component of the Goldstone superfield part of the Nöether gauge current vector superfield [31]

$$\begin{aligned} J_A(x, \theta, \bar{\theta}) &= \bar{H}_T T_T^A H_T + \bar{H}_B T_B^A H_B \\ &= \left\{ -\frac{i}{2} A_A^i(\vec{\pi}) \frac{\partial}{\partial \pi^i} + \frac{i}{2} \bar{A}_A^i(\vec{\pi}) \frac{\partial}{\partial \bar{\pi}^i} \right\} \overbrace{[\bar{H}_T H_T + \bar{H}_B H_B]}^{=K}. \end{aligned} \quad (\text{B.34})$$

Thus the Killing potentials become

$$J_A = J(x, 0, 0) = A_T^\dagger T_T^A A_T + A_B^\dagger T_B^A A_B$$

$$\begin{aligned}
&= -\frac{i}{2} \left(A_A^i \frac{\partial}{\partial A^i} - \bar{A}_A^i \frac{\partial}{\partial A^{\dagger i}} \right) [A_T^\dagger A_T + A_B^\dagger A_B] \\
&= -i \left(A_A^i \frac{\partial}{\partial A^i} - \bar{A}_A^i \frac{\partial}{\partial A^{\dagger i}} \right) [A_\sigma^\dagger A_\sigma + \vec{A}^\dagger \cdot \vec{A}] \\
&= -i \left(A_A^i \frac{\partial}{\partial A^i} - \bar{A}_A^i \frac{\partial}{\partial A^{\dagger i}} \right) \left[\sqrt{\frac{1}{2} v_T v_B - \vec{A}^{\dagger 2}} \sqrt{\frac{1}{2} v_T v_B - \vec{A}^2} + \vec{A}^\dagger \cdot \vec{A} \right] \\
&= i \left\{ A^i \bar{A}_A^i - A^{i\dagger} A_A^i + A^i A_A^i \frac{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^{\dagger 2}}}{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^2}} - A^{i\dagger} \bar{A}_A^i \frac{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^2}}{\sqrt{\frac{1}{2} v_T v_B - \vec{A}^{\dagger 2}}} \right\}.
\end{aligned} \tag{B.35}$$

Differentiating the superfield currents J_A with respect to the Goldstone superfields, we find the superfield differential equations relating the Killing vectors and the Killing potentials (all superfields here)

$$\begin{aligned}
\frac{\partial}{\partial \pi^i} J_A &= i \bar{A}_A^j g_{\bar{j}i} \\
\frac{\partial}{\partial \bar{\pi}^i} J_A &= -i A_A^j g_{ij}.
\end{aligned} \tag{B.36}$$

Appendix C Coordinates And Killing Vectors

The $SU(2) \times U(1)$ gauge transformations of the 2×2 matrix chiral superfield U ,

$$U' = e^{i\vec{T} \cdot \vec{\Lambda}} U e^{-iT^3 \Lambda_Y}, \quad (\text{C.1})$$

induce general coordinate transformations of the Goldstone superfields used to parameterize the Kähler manifold. For the “sigma model” choice of coordinates in the body of the paper,

$$U = \sigma \mathbf{1} + 2i\vec{T} \cdot \vec{\pi}, \quad (\text{C.2})$$

the infinitesimal gauge transformations yield

$$\begin{aligned} \delta\pi^1 &= +\Lambda^1\sigma - \Lambda^2\pi^3 + \Lambda^3\pi^2 + \frac{1}{2}\Lambda_Y\pi^2 \\ \delta\pi^2 &= +\Lambda^1\pi^3 + \Lambda^2\sigma - \Lambda^3\pi^1 - \frac{1}{2}\Lambda_Y\pi^1 \\ \delta\pi^3 &= -\Lambda^1\pi^2 + \Lambda^2\pi^1 + \Lambda^3\sigma - \frac{1}{2}\Lambda_Y\sigma, \end{aligned} \quad (\text{C.3})$$

along with the constrained field’s variation

$$\delta\sigma = -\vec{\Lambda} \cdot \vec{\pi} + \frac{1}{2}\Lambda_Y\pi^3. \quad (\text{C.4})$$

(This also could be obtained directly from the gauge transformations of H_T and H_B , equation (B.20).) Indeed, since σ is a constrained field, $\det U \equiv \frac{1}{2}v_T v_B$, the Goldstone multiplets transform non-linearly upon substitution of $\sigma = \sqrt{\frac{1}{2}v_T v_B - \vec{\pi}^2}$ into equation (C.3) and the σ variation follows from their transformations applied to the constraint equation (C.4).

The chiral superfield Killing vectors, $A_A^i(\vec{\pi})$, define these non-linear transformations (here let $\Lambda^4 = \frac{1}{2}\Lambda_Y$)

$$\delta\pi^i = \Lambda^A \delta_A \pi^i \equiv \Lambda^A A_A^i(\vec{\pi}). \quad (\text{C.5})$$

Thus we secure

$$A_A^i(\vec{\pi}) = \begin{cases} -\epsilon^{iak}\pi^k + \delta_a^i\sigma & \text{for } A = a = 1, 2, 3 \\ -\epsilon^{i3k}\pi^k - \delta_3^i\sigma & \text{for } A = 4 \end{cases}. \quad (\text{C.6})$$

The Killing potential equation, (B.36), is satisfied by A_A^i .

The “standard” coordinates are defined by the exponential representation of the 2×2 matrix chiral superfield U [26]

$$U \equiv f e^{2i\vec{T} \cdot \vec{\xi}} = f \left[\cos \sqrt{\vec{\xi}^2} + 2i\vec{T} \cdot \vec{\xi} \frac{\sin \sqrt{\vec{\xi}^2}}{\sqrt{\vec{\xi}^2}} \right], \quad (\text{C.7})$$

with

$$\det U = f^2 = \frac{1}{2}v_T v_B \quad (\text{C.8})$$

and ξ^i the three Goldstone boson superfields. The transformation to the σ -model coordinates, π^i , follows from the equality of the respective representations for U

$$\pi^i = \frac{\sin \sqrt{\vec{\xi}^2}}{\sqrt{\vec{\xi}^2}} f \xi^i. \quad (\text{C.9})$$

The constraint equation equality, $f \cos \sqrt{\vec{\xi}^2} = \sigma = \sqrt{\frac{1}{2}v_T v_B - \vec{\pi}^2}$, follows from equation (C.2). Inverting the equation yields ξ^i as a function of π^i

$$\xi^i = \rho(\vec{\pi}^2) \pi^i, \quad (\text{C.10})$$

with

$$\rho(\vec{\pi}^2) = \frac{\arcsin \sqrt{\frac{\vec{\pi}^2}{f^2}}}{f \sqrt{\frac{\vec{\pi}^2}{f^2}}}. \quad (\text{C.11})$$

From simple geometry, note that $\sin \sqrt{\xi^2} = \sqrt{\frac{\pi^2}{f^2}}$, so that the $\rho\sigma$ product is simply

$$\rho\sigma = f\sqrt{\xi^2} \cot \sqrt{\xi^2}. \quad (\text{C.12})$$

The non-linear gauge transformations of the standard coordinates are

$$\begin{aligned} \delta\xi^i &= \frac{\partial\xi^i}{\partial\pi^j} \delta\pi^j \equiv \Lambda^A P_j^i(\vec{\xi}) A_A^j(\vec{\xi}/\rho) \\ &= \left[\delta^{ij} - \left(1 - \frac{1}{\rho\sigma}\right) \frac{\xi^i \xi^j}{\xi^2} \right] \rho A_A^j(\vec{\xi}/\rho). \end{aligned} \quad (\text{C.13})$$

Exploiting the coordinate transformations, $\xi^i = \rho\pi^i$, $\sigma = f \cos \sqrt{\xi^2}$ and equation (C.12), the standard coordinate Killing vectors, $X_A^i(\vec{\xi})$, are secured

$$\delta\xi^i \equiv \Lambda^A X_A^i(\vec{\xi}), \quad (\text{C.14})$$

where $\rho = \sqrt{\xi^2} \csc \sqrt{\xi^2}$ and

$$\begin{aligned} X_A^i(\vec{\xi}) &= P_j^i(\vec{\xi}) A_A^j(\vec{\xi}/\rho) \\ &= \begin{cases} -\epsilon^{iak} \xi^k + \frac{\xi^i \xi^a}{\xi^2} + \left(\delta_a^i - \frac{\xi^i \xi^a}{\xi^2} \right) \sqrt{\xi^2} \cot \sqrt{\xi^2} & \text{for } A = a = 1, 2, 3 \\ -\epsilon^{i3k} \xi^k - \frac{\xi^i \xi^3}{\xi^2} - \left(\delta_3^i - \frac{\xi^i \xi^3}{\xi^2} \right) \sqrt{\xi^2} \cot \sqrt{\xi^2} & \text{for } A = 4. \end{cases} \end{aligned} \quad (\text{C.15})$$

The simple Kähler potential can be written as

$$K = \text{Tr}[\bar{U}U] = f^2 \text{Tr} \left[e^{-i\vec{\xi} \cdot \vec{T}} e^{i\vec{\xi} \cdot \vec{T}} \right]. \quad (\text{C.16})$$

The Kähler manifold metric can be obtained from the potential

$$g_{i\bar{j}}(\vec{\xi}, \vec{\xi}) = \frac{\partial^2 K}{\partial \xi^i \partial \bar{\xi}^j}. \quad (\text{C.17})$$

The gauge invariant potential is given by

$$K = \text{Tr} \left[\bar{U} e^{-2g_2 \vec{T} \cdot \vec{W}} U e^{2g_1 T^3 Y} \right]. \quad (\text{C.18})$$

The superfield Killing potentials, J^A , are the Noether gauge currents found from the auxiliary field D -term coupling in K

$$J^A = \begin{cases} -2g_2 \text{Tr} [\bar{U} T^a U] & \text{for } A = a = 1, 2, 3 \\ +2g_1 \text{Tr} [\bar{U} U T^3] & \text{for } A = 4 \end{cases}. \quad (\text{C.19})$$

As in appendix B, the Killing potential equations are valid by construction

$$\begin{aligned} \frac{\partial}{\partial \xi^i} J_A &= i \bar{X}_A^j g_{\bar{j}i} \\ \frac{\partial}{\partial \bar{\xi}^i} J_A &= -i X_A^j g_{i\bar{j}}. \end{aligned} \quad (\text{C.20})$$

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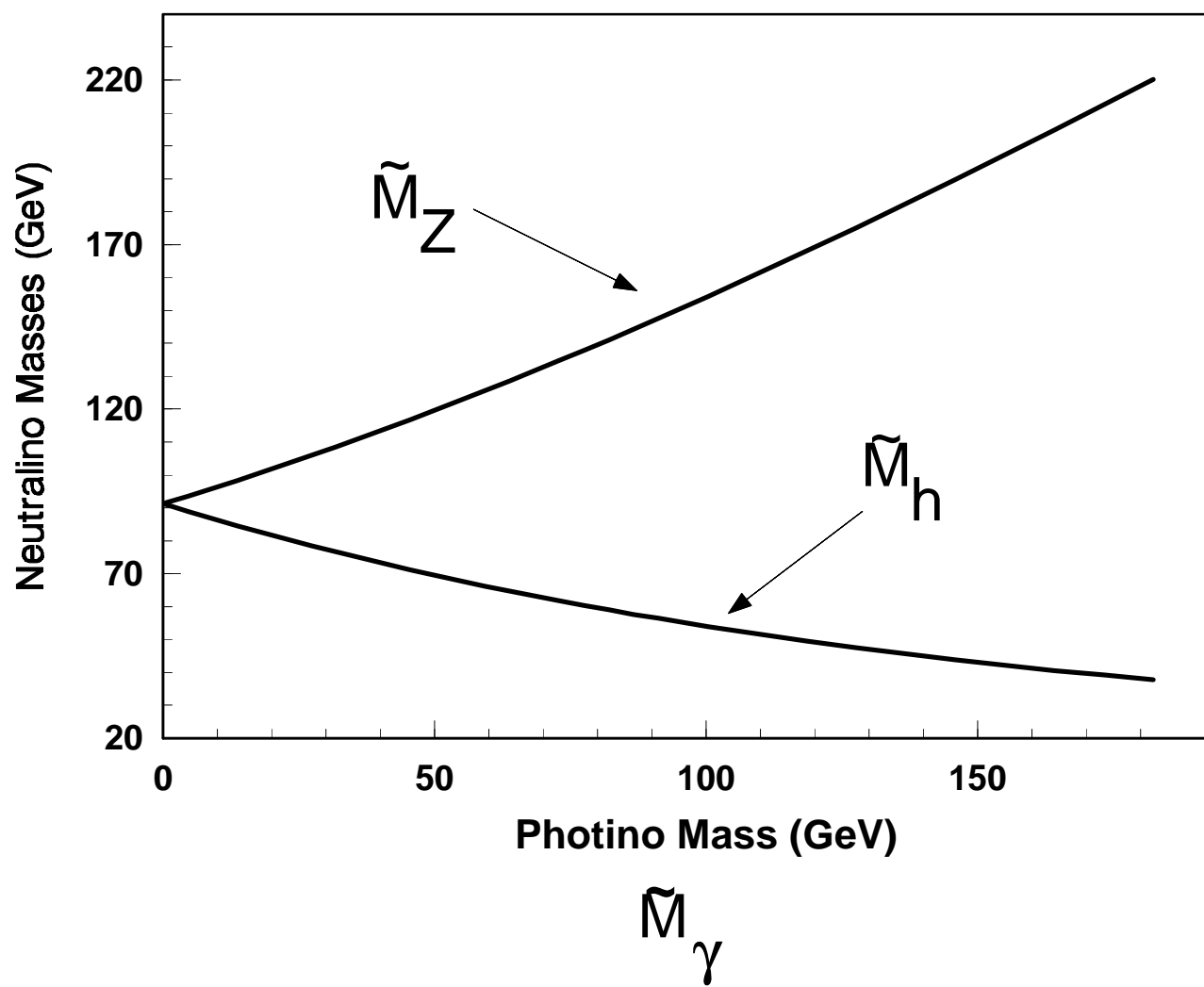
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